Constrained mixed-integer blackbox optimization for the selection of materials of an automotive vehicle

Marie-Ange Dahito

Supervisors:
Laurent Genest[1] *(COVP.Optim)*
Nikolaus Hansen[2,3], Dimo Brockhoff[2,3] *(CMAP, RandOpt)*
José Neto[4], Alessandro Maddaloni[4] *(Méthodes)*

1 STELLANTIS 2 Inria 3 ÉCOLE POLYTECHNIQUE UNIVERSITÉ PARIS-SACLAY 4 TELECOM SudParis IP PARIS

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Summary

- More and more restrictive regulations (safe automation, CO_2 emissions...)
- Stellantis committed to a reduction of its consumption
  $\Rightarrow$ **Reductions of the weights of the vehicles**
- Rising costs (new embedded technologies, electrification, mileage capacity...)
- Need to meet a certain performance while maintaining low production costs
  $\Rightarrow$ Numerical optimization used as a decision making tool at different phases in the vehicle design process
  $\Rightarrow$ **Optimization on the body in white** through finite element models
- e.g. optimization on the body in white to determine the optimal thicknesses

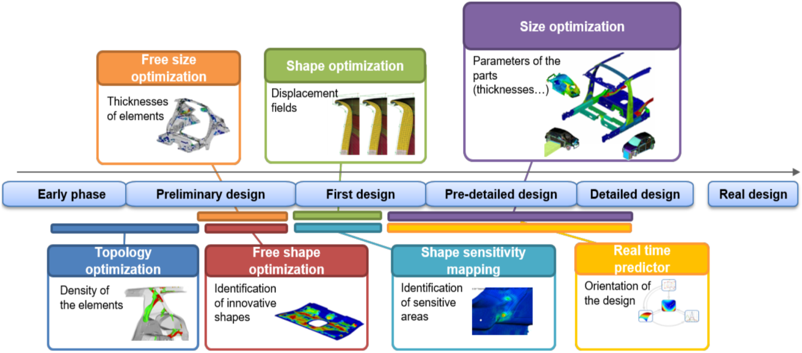
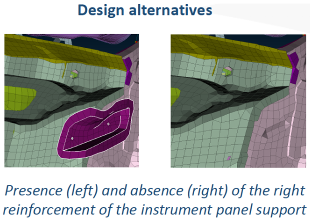


Figure: Numerical optimization at Stellantis during the vehicle design process.

- A lower weight of a part with the same property ⇒ more expensive material
- A better balance between costs and weights reductions has to be found
- Idea: **add new optimization levers** in the size optimization
  ⇒ The choice of materials *(cf right picture)*
  ⇒ The design alternatives *(cf left picture)*
- **No available algorithm** to fulfill these needs within an acceptable numerical budget
  ⇒ Need to design a new algorithm: goal of the PhD



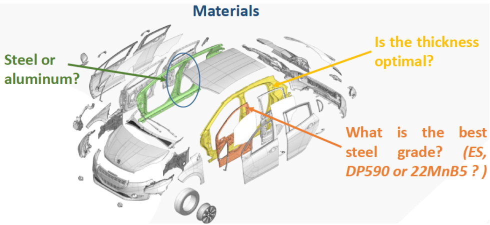


Figure: The design alternatives and the choice of materials as optimization levers.

- The variables are the geometric parameters (thicknesses, shape parameters) of the model **and the materials** of each part of the body in white (BIW)
- **Minimize the cost** of the BIW
- **Minimize the weight** of the BIW
- Maximize the carry-over (reused parts)
- Respect the expected performance of the vehicle
- Long simulation times from finite element models
  $\rightarrow$ **Stiffness**: 20 min to 1 hour
  $\rightarrow$ **Crashworthiness**: 6 to 8 hours
  $\rightarrow$ **Vibro-acoustic** (NVH): 1 to 2 hours
- The numerical cost is important as a solution is desired in a limited time

Figure: Stiffness, crashworthiness and vibro-acoustic performance from finite element models.

- No analytical formula of the finite element models –> **blackbox** optimization
- Several objectives –> **multi-objective** optimization
- Materials cannot be ranked: categorical variables –> **mixed-integer** optimization
- Up to 200 constraints to satisfy –> **constrained** optimization
- Limited computation capacity

Separately, each of these optimization branches has a quite furnished literature.

But a complexity lies in their overlap and the fact that the corresponding literature is relatively poor.

- Evolutionary algorithms
  - NSGA-II: well-known genetic algorithm for multi-objective optimization
  - CMA-ES: state-of-the-art evolutionary algorithm for derivative-free optimization
- Direct local search methods
  - MADS: well-known method using asymptotically dense directions

$\rightarrow$ *The methods above use numerous evaluations*

- Surrogate-based techniques (kriging, radial basis functions...) can be used to save blackbox evaluations

→ The real case is computationally expensive and a smaller version was not available

- Finite element mechanical test case
- 8 nodes, 13 elements of square sections
  - **Clamped to both sides** at Nodes 1 and 5
  - Application of a **vertical force** at Node 3
- Three possible objective functions: cost, weight and compliance
- Mixed-integer problem (variables: materials and thicknesses)
- Enable to cope with the long computation times
- Tests of NSGA-II and CMA-ES

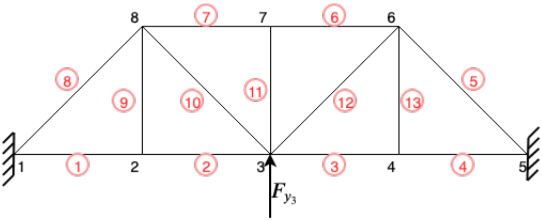


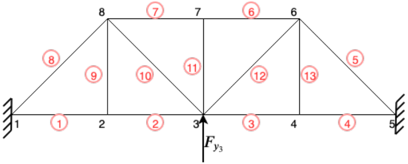
Figure: Finite element mechanical test case of bar elements.

Example of a single-objective problem

$m \in [\![1, 4]\!]$ (1 for titanium, 2 for magnesium, 3 for steel and 4 for aluminum),
t: *thickness*, U: *displacement*

$$\underset{x \in \mathbb{R}^{26}}{\text{minimize}} \quad \text{cost}(x) \quad x = [m_{\text{el}1}, \dots, m_{\text{el}13}, t_{\text{el}1}, \dots, t_{\text{el}13}]$$

$$\text{subject to} \quad \begin{cases} |u_{y3}(x)| < u_{y3,\max} \\ x_{1:13} \in [\![1, 4]\!] \quad \textit{discrete parameters} \\ x_{14:26} \in [0.01, 0.05] \text{ (m)}, \end{cases}$$



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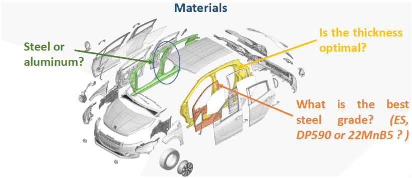

Figure: The choice of materials as an optimization lever.

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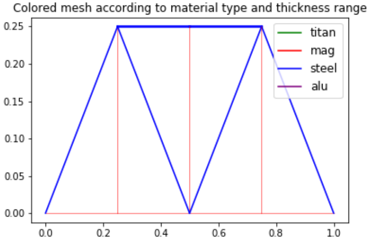

Figure: Mesh solution found by NSGA-II for weight optimization. Corresponding cost ≈ 6.89€.

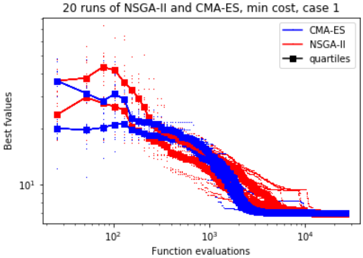

Figure: Evolution of the costs for NSGA-II and CMA-ES with thick lines for the quartiles.

Example of a multi-objective problem

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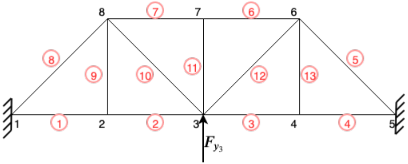
t: *thickness*, U: *displacement*

$$\underset{x \in \mathbb{R}^{26}}{\text{minimize}} \quad \begin{cases} \text{cost}(x) & x = [m_{\text{el}1}, \ldots, m_{\text{el}13}, t_{\text{el}1}, \ldots, t_{\text{el}13}] \\ \text{weight}(x) \\ \text{compliance}(x) \end{cases}$$

$$\text{subject to} \quad \begin{cases} |u_{y3}(x)| < u_{y3,\max} \\ x_{1:13} \in [\![1, 4]\!] & \textit{discrete parameters} \\ x_{14:26} \in [0.01, 0.05] \text{ (m)}, \end{cases}$$



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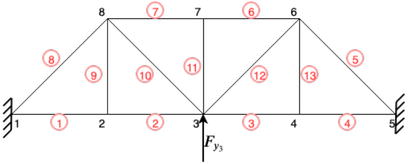


Figure: Finite element mechanical test case of bar elements.

nsga2, rounded offspring, min cost&weight&compliance, case 1

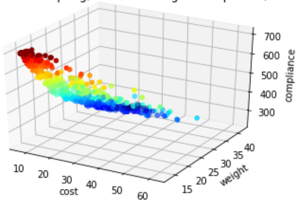


Figure: Pareto estimations for 20 runs of NSGA-II.

Comments from these numerical tests

- CMA-ES seems to converge faster than NSGA-II
- NSGA-II sometimes ends to a smaller objective
- The Pareto estimations of NSGA-II cover varied zones according to the run
- Many evaluations needed before convergence (between 10^3 and 10^4)
- Finding the good penalization can be laborious even for a single non-linear constraint

- Use of the continuous suite BBOB of the COCO platform
- Benchmarking of solvers of the library SciPy

  → Co-written workshop paper for the GECCO conference of 2019

  → SLSQP performs well on BBOB

  But further tests on mixed integer problems were less successful ⇒ not kept

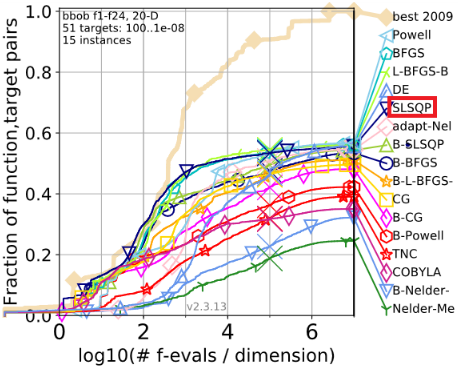


Figure: ECDF plot: performance of multivariate solvers of SciPy on BBOB, aggregated in dimension 20.

- Tests of MADS from the Nomad software on BBOB
  → **The variants ORTHO $N+1$ NEG and ORTHO 2N of MADS perform better than the other ORTHO settings**

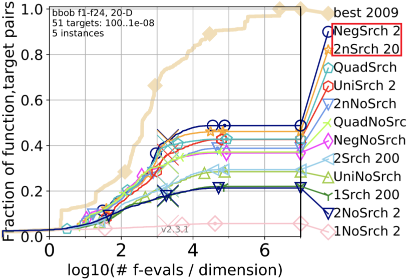
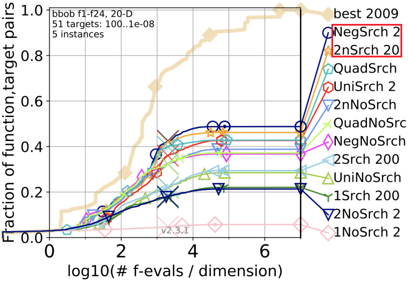


Figure: ECDF plot: performance of the OrthoMADS algorithms on BBOB, aggregated in dimension 20.

- Tests of MADS from the Nomad software on BBOB
  → The variants ORTHO $\mathbf{N+1}$ NEG and ORTHO **2N** of MADS perform better than the other ORTHO settings
  → **Comparison with other algorithms: MADS in the mean and CMA-ES among the best on the test problems**



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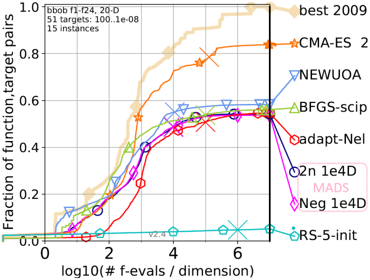


Figure: ECDF plot: performance of a few algorithms on BBOB, aggregated in dimension 20.

- Tests of MADS from the Nomad software on BBOB
  → The variants ORTHO $\mathbf{N+1}$ NEG and ORTHO $\mathbf{2N}$ of MADS perform better than the other ORTHO settings
  → Comparison with other algorithms: MADS in the mean and CMA-ES among the best on the test problems
  → **MADS performs well on some multi-modal problems like the Gallagher functions with several local optima**

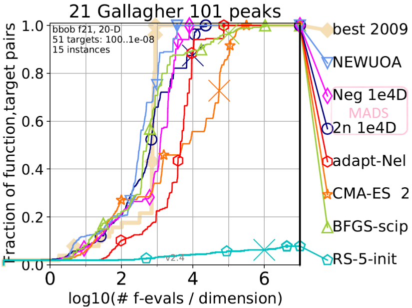


Figure: ECDF plot: performance of a few algorithms on the Gallagher 101 peaks function, aggregated in dimension 20.

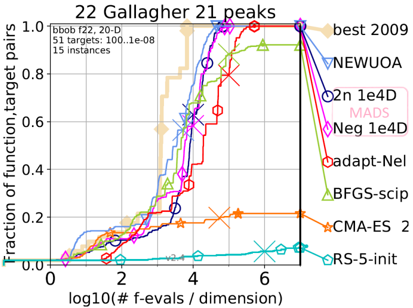


Figure: ECDF plot: performance of a few algorithms on the Gallagher 21 peaks function, aggregated in dimension 20.

- Test the variants ORTHO $N+1$ NEG and ORTHO $2N$ of MADS on constrained and mixed-integer suites
- Write a paper on the performance of MADS
- Test promising methods on a small automotive test case: a lateral crashworthiness case consisting of 6 parts on the battery zone
- First focus on single-objective optimization

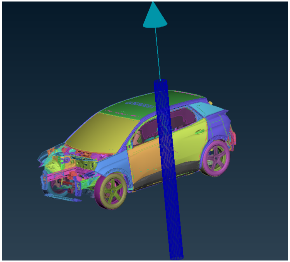


Figure: Pole lateral crash.

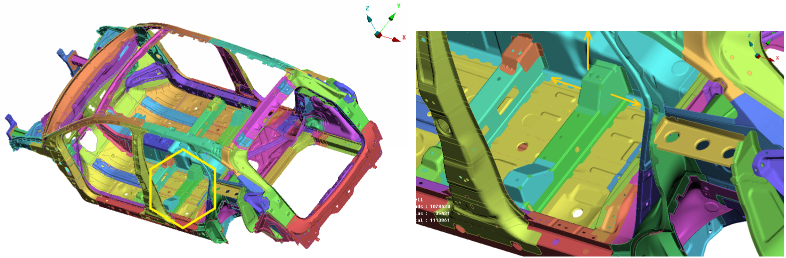


Figure: CAD model of a vehicle showing the underseat cross member on the battery zone.

The optimization problem is:

$$\min_{x \in \mathbb{D}} \quad \text{weight} \qquad \text{(kg)}$$

$$\text{s.t.} \quad \begin{cases} \text{cost} & \leq \text{initial cost} & (€) \\ \text{car intrusion} & \leq \text{maximal intrusion} & (\text{mm}) \\ \text{strength on the battery} & \leq \text{maximal strength} & (\text{kN}) \\ \text{4 decelerations on the battery} & \leq \text{4 maximal decelerations} & (\text{m/s}^2) \\ \text{displacement} & \leq \text{maximal displacement} & (\text{mm}) \\ s_1 \in [-10, 0], s_2 \in [-30, 30], & s_3 \in [-20, 20] \text{ and } s_4 \in [0, 20] & (\text{mm}) \\ t_i \in [0.65, 2], & i \in \{1, .., 6\} & (\text{mm}) \\ m_i \in \{1, .., 11\}, & i \in \{1, .., 6\}, & \end{cases}$$

with $x = [s_1, .., s_4, t_1, .., t_6, m_1, .., m_6]$ (shape parameters, thicknesses and materials).

- Comparison of existing methods on problems stemming from the literature and applications:
  - Variants of ORTHOMADS
  - Deterministic algorithms (NEWUOA, BFGS, Nelder-Mead method...)
  - Evolutionary algorithms (CMA-ES, NSGA-II, PSO...)
- Development of new model-based methods for (constrained) mixed-integer problems
  - Survey and evaluation of surrogate models (Kriging, RBF, RSM...) on mixed-integer literature and application problems
  - Development of new approaches based on different types of surrogates to deal with the categorical variables
  - Comparison of the new proposals with deterministic methods and evolutionary algorithms on:
    - literature and application problems
    - an automotive problem

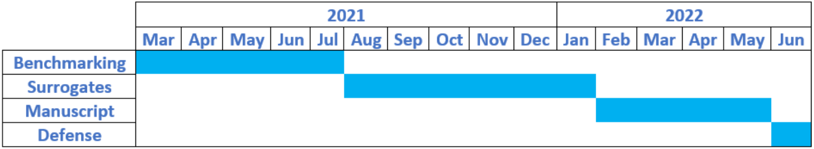
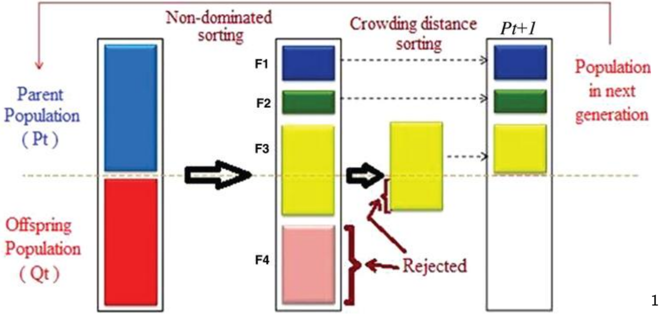


Figure: Schedule for the rest of the PhD.

Thank you for your attention!

NSGA-II

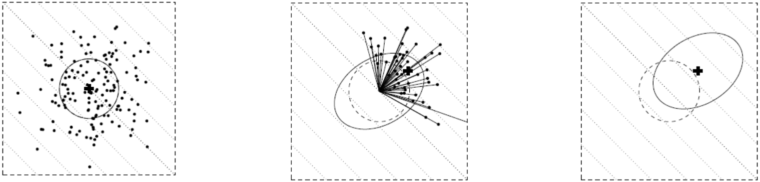


[1]

Figure: The generation of populations in NSGA-II through recombination, non-dominated sorting and crowding distance sorting.

[1] E. Abiri, Z. Bezareh, and A. Darabi. The optimum design of RAM cell based on the modified-GDI method using Non-dominated Sorting Genetic Algorithm II (NSGA-II). *Journal of Intelligent & Fuzzy Systems*, 32(6):4095–4108, 2017.

CMA-ES

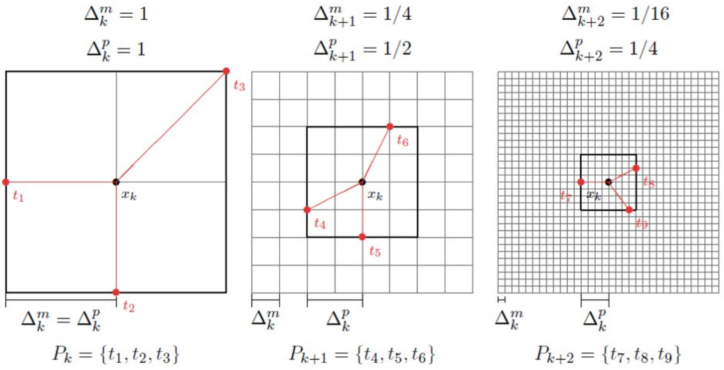


$x_i = m_{\text{old}} + y_i, \quad y_i \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$ $\quad \mathbf{C} \leftarrow \frac{1}{\mu} \sum (x_{i:\lambda} - m_{\text{old}})(x_{i:\lambda} - m_{\text{old}})^{\mathrm{T}}$ $\quad m_{\text{new}} = m_{\text{old}} + \frac{1}{\mu} \sum y_{i:\lambda}$ [2]

Figure: CMA-ES: Sampling of the population (left), update of the covariance matrix from the best individuals (middle) and update of the mean of the next generation (right).

[2] Y. Akimoto and N. Hansen. CMA-ES and Advanced Adaptation Mechanisms. In *Proceedings of the Genetic and Evolutionary Computation Conference Companion*, GECCO '18, page 720–744, New York, NY, USA, 2018. Association for Computing Machinery.

MADS

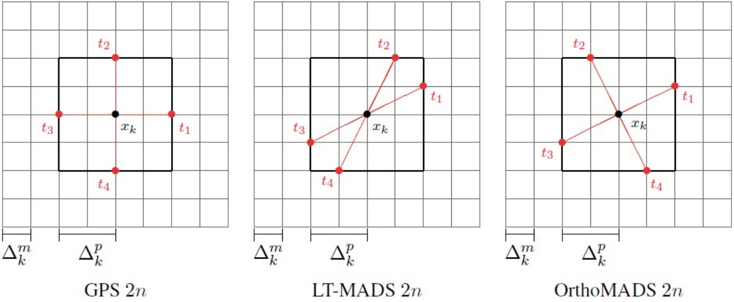


[3]

Figure: Example of mesh adaptation and directions generation in MADS.

[3] S. Le Digabel. Algorithm 909: NOMAD: Nonlinear optimization with the MADS algorithm. *ACM Trans. Math. Softw.*, 37:44:1–44:15, 2011.

MADS directions



[4]

Figure: Three families of directions for the poll step of MADS.

[4] S. Le Digabel. Algorithm 909: NOMAD: Nonlinear optimization with the MADS algorithm. *ACM Trans. Math. Softw.*, 37:44:1–44:15, 2011.

Surrogate-based optimization

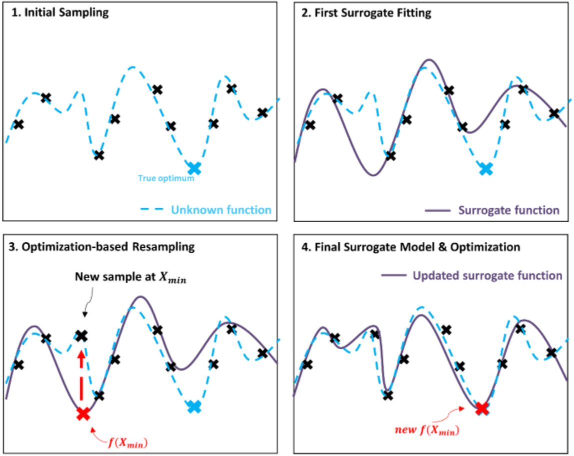
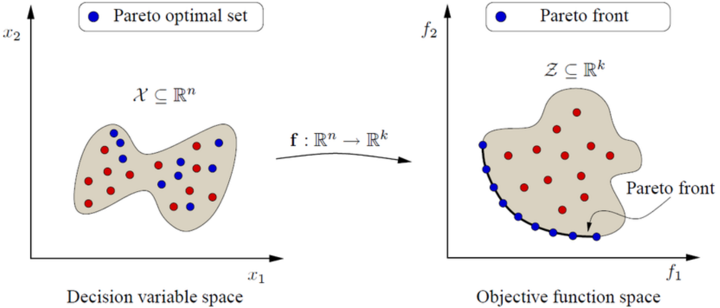


Figure: An example of surrogate-based optimization. Source: Kim, S.H., Boukouvala, F. Machine learning-based surrogate modeling for data-driven optimization: a comparison of subset selection for regression techniques. Optim Lett 14, 989–1010 (2020).

Pareto dominance



[5]

Figure: Pareto dominance and Pareto front.

[5] M. H. Muaafa. *Multi-criteria Decision-making Framework for Surveillance and Logistics Applications*. Diss. Stevens Institute of Technology, 2015.

Cost optimization with the 3 methods

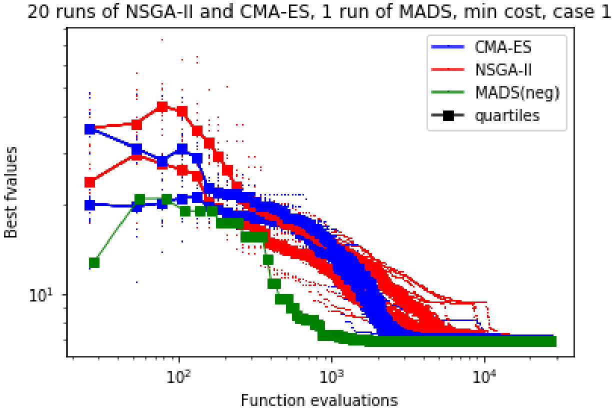


Figure: Evolution of the costs for 20 runs of CMA-ES (blue) and NSGA-II (red) and 1 run of MADS (green) with thick lines for the quartiles.

Cost optimization with NSGA-II and two population sizes

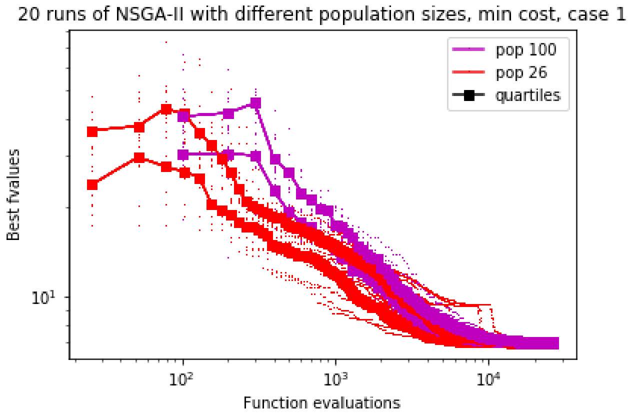


Figure: Evolution of the costs for 20 runs of NSGA-II with a population size of 26 (red) and 100 (mauve) with thick lines for the quartiles.

Cost optimization with CMA-ES and two population sizes

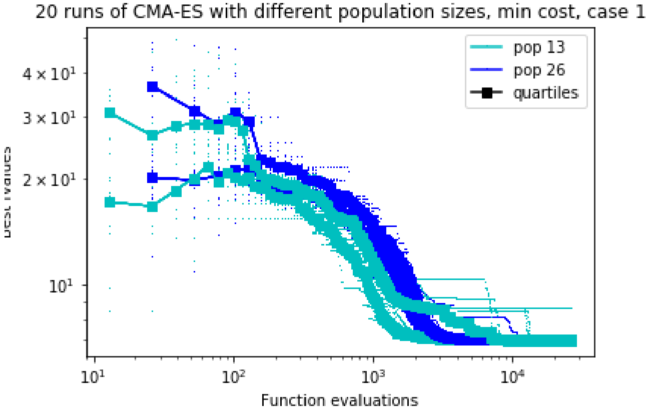


Figure: Evolution of the costs for 20 runs of CMA-ES with a population size of 26 (blue) and 13 (cyan) with thick lines for the quartiles.